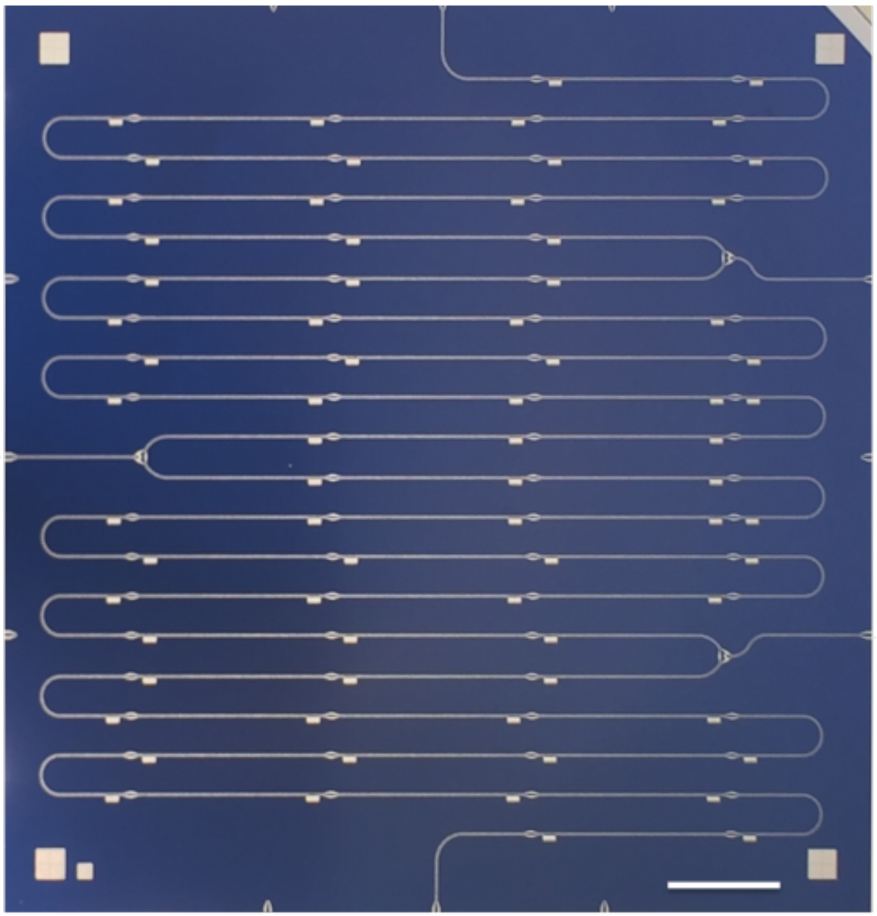
Dissipative phase transition in a one-dimensional circuit QED lattice: Theory

Andy C. Y. Li



Mattias Fitzpatrick
(Princeton U)

Neereja Sundaresan
(Princeton U)

Andrew Houck
(Princeton U)

Jens Koch
(Northwestern U)

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UNIVERSITY

1D Circuit-QED chain - model

Resonators coupled to transmons

$$\mathrm{H}_{\text{on-site}} = \sum_{j=1}^{N} \left[\omega \mathrm{a}_j^\dagger \mathrm{a}_j + \mathrm{H}_{TQ;j} \right]$$

Hopping between resonators

$$\mathrm{H}_{\text{hop}} = \kappa \sum_{j=1}^{N-1} \left(\mathrm{a}_j^\dagger \mathrm{a}_{j+1} + \mathrm{a}_j \mathrm{a}_{j+1}^\dagger \right)$$

Drive at the input port

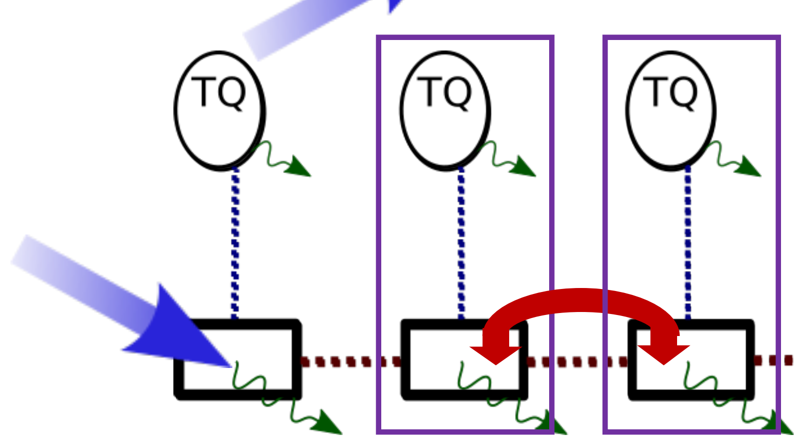
$$\mathrm{H}_{\text{drive}} = \varepsilon \left(\mathrm{a}_1^\dagger e^{i\omega_d t} + \text{h.c.} \right)$$

Master equation

$$\frac{d\rho}{dt} = -i[\mathrm{H}_{\text{on-site}} + \mathrm{H}_{\text{hop}} + \mathrm{H}_{\text{drive}}, \rho] + \gamma \sum_{j=1}^{N} \mathbb{D}[\mathrm{a}_j]\rho + \Gamma \sum_{j=1}^{N} \mathbb{D}_{TQ;j}\rho$$

Transmon qubit relaxation

photon decay

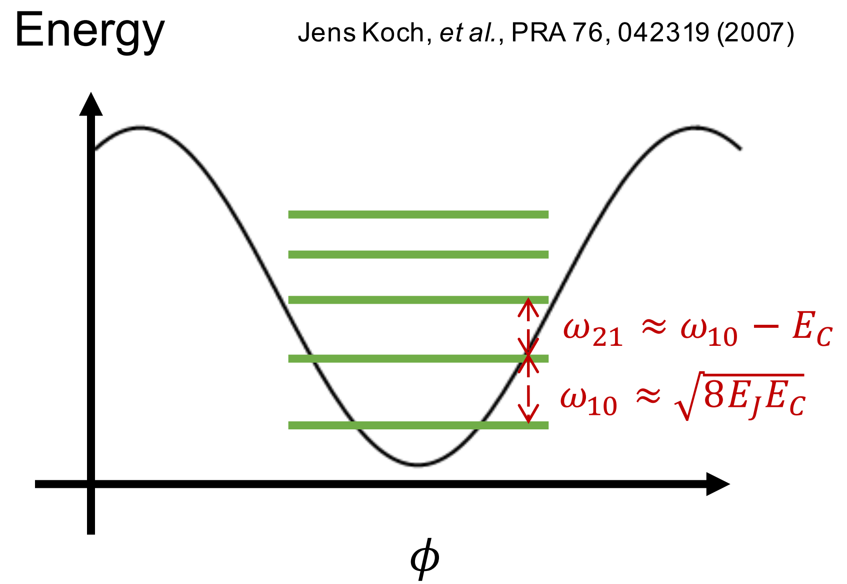


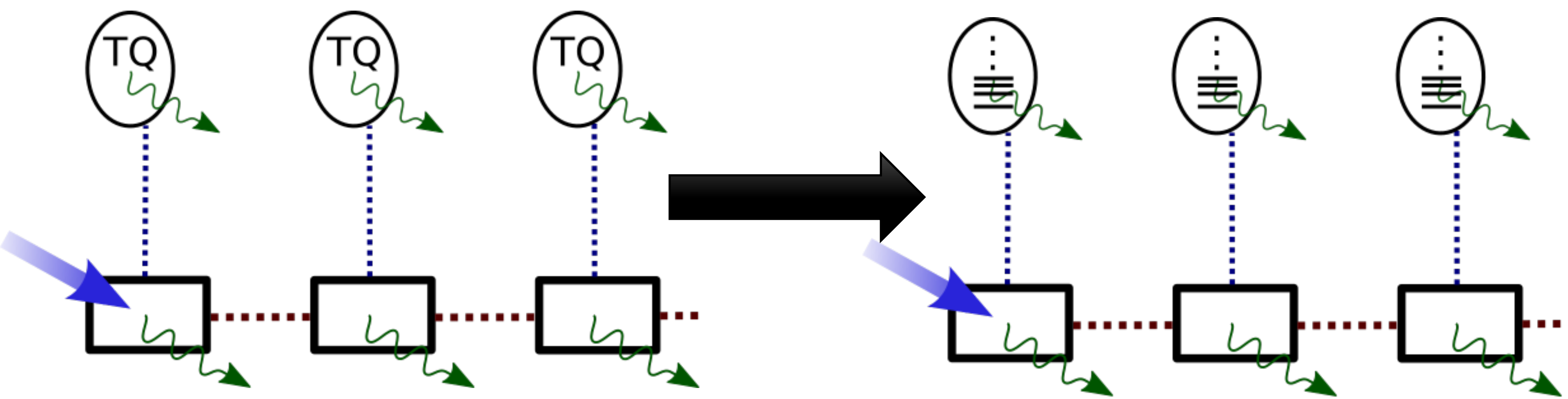
Transmon ≈ Bose-Hubbard

$$H_{TQ} = 4E_C\left(\frac{q}{2e} - \frac{q_g}{2e}\right)^2 - E_J\cos\left(\frac{2e}{\hbar}\phi\right)$$

$$H_{BH} = \underset{\omega_{10}}{\underline{\Omega}}\, b^\dagger b + \underset{-E_C/2}{\underline{U}}\, b^\dagger b^\dagger b b$$

Jens Koch, *et al.*, PRA 76, 042319 (2007)





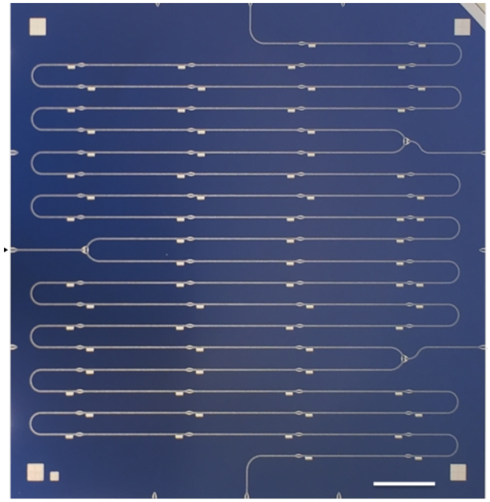
Theoretical challenges

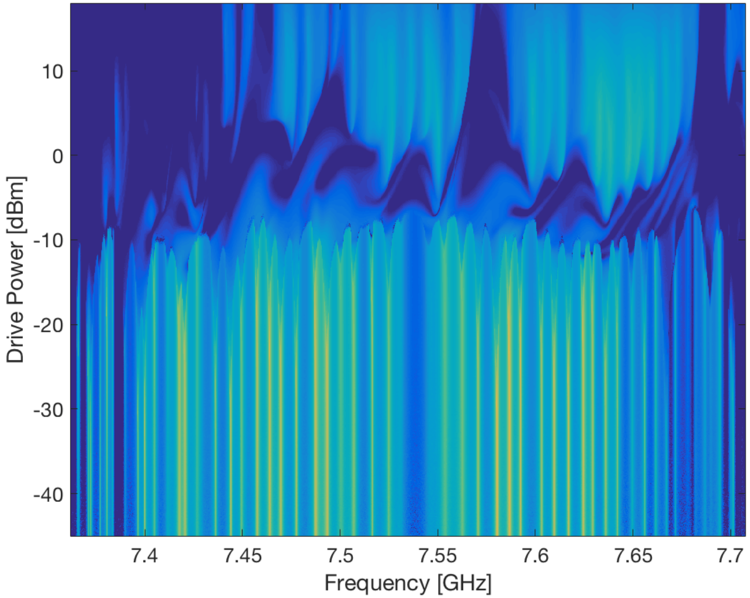
- Strong coupling: $g \approx 500$ MHz (Detuning $\Delta \approx 2$ GHz)

- Drive power over 5 orders of magnitude: ~~dispersive approximation~~

- Large number of sites: ~~direct numeric~~

- Critical behavior: ~~perturbative treatment~~



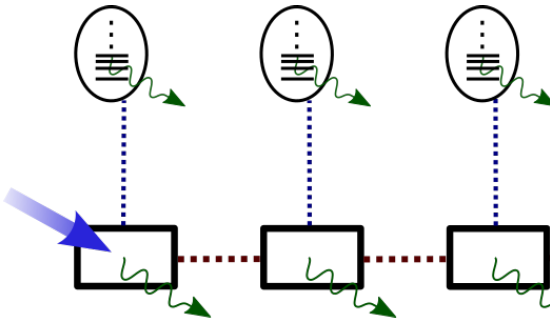


Mean-field treatment

$$\frac{\mathrm{d}\rho}{\mathrm{d}t} = \mathbb{L}\rho - i\left[\mathsf{H}, \rho\right] + \gamma \sum_{j=1}^{N} \mathbb{D}\left[\mathsf{a}_j\right]\rho + \Gamma \sum_{j=1}^{N} \mathbb{D}\left[\mathsf{b}_j\right]\rho$$

$$\mathsf{H} = \sum_{j=1}^{N} \left[\delta\omega\, \mathsf{a}_j^\dagger \mathsf{a}_j + \delta\Omega_j\, \mathsf{b}_j^\dagger \mathsf{b}_j + U \mathsf{b}_j^\dagger \mathsf{b}_j^\dagger \mathsf{b}_j \mathsf{b}_j + g\left(\mathsf{a}_j^\dagger \mathsf{b}_j + \mathsf{a}_j \mathsf{b}_j^\dagger\right)\right]$$

$$+ \sum_{j=1}^{N-1} \kappa \left(\mathsf{a}_j^\dagger \mathsf{a}_{j+1} + \mathsf{a}_j \mathsf{a}_{j+1}^\dagger\right) + \epsilon\left(\mathsf{a}_1^\dagger + \mathsf{a}_1\right)$$



Mean-field approximation:

Small fluctuation

$$\mathsf{a}_j = \alpha_j + \mathsf{c}_j \qquad \mathsf{b}_j = \beta_j + \mathsf{d}_j$$

Mean-field dynamics equations

144 coupled nonlinear ODEs

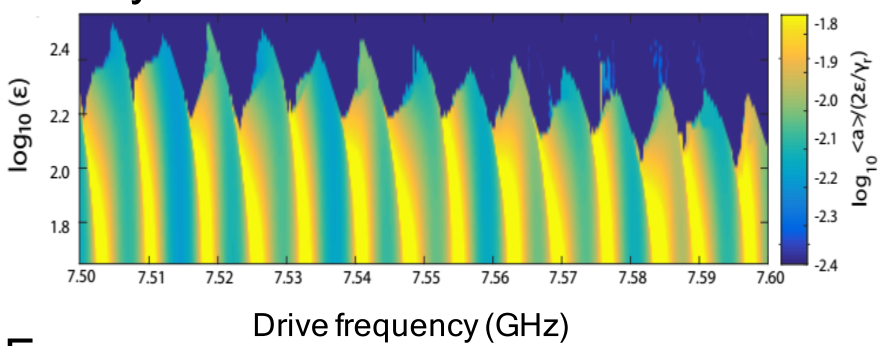
$$i\frac{\mathrm{d}\alpha_j}{\mathrm{d}t} = \left(\delta\omega - i\frac{\gamma}{2}\right)\alpha_j + g\beta_j + \kappa\left(\alpha_{j-1} + \alpha_{j+1}\right) + \epsilon\delta_{j1}$$

$$i\frac{\mathrm{d}\beta_j}{\mathrm{d}t} = \left(\delta\Omega_j - i\frac{\Gamma}{2}\right)\beta_j + g\alpha_j + 2U\left|\beta_j\right|^2\beta_j$$

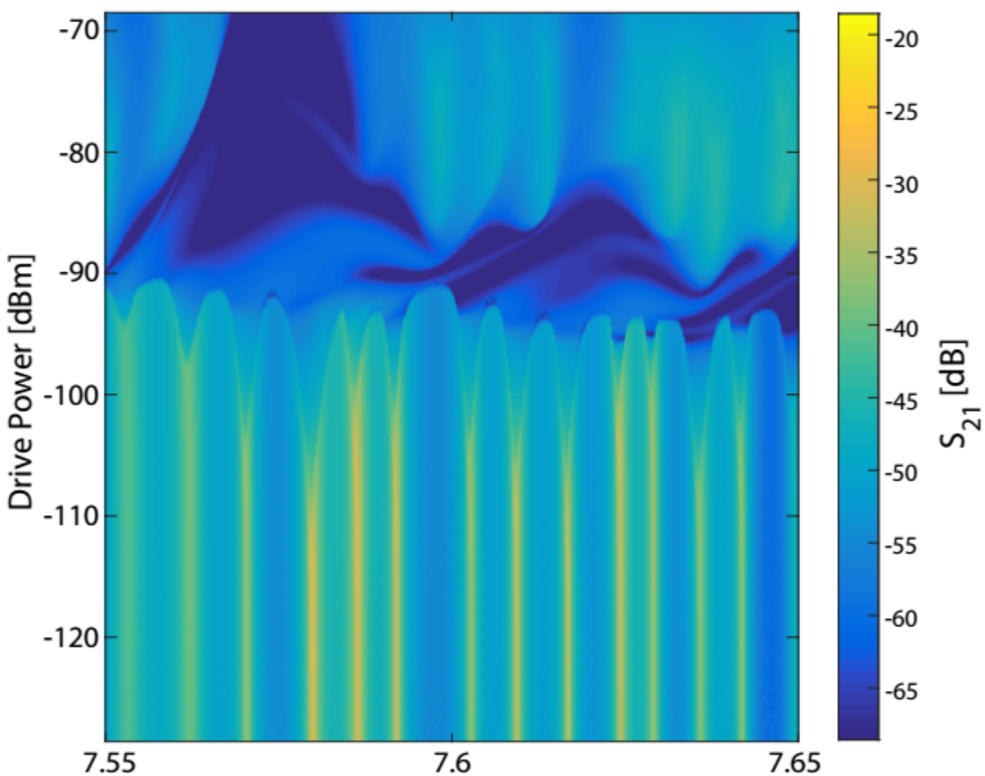
Evolve for a long time

Transmission: theory vs. exp.

Theory

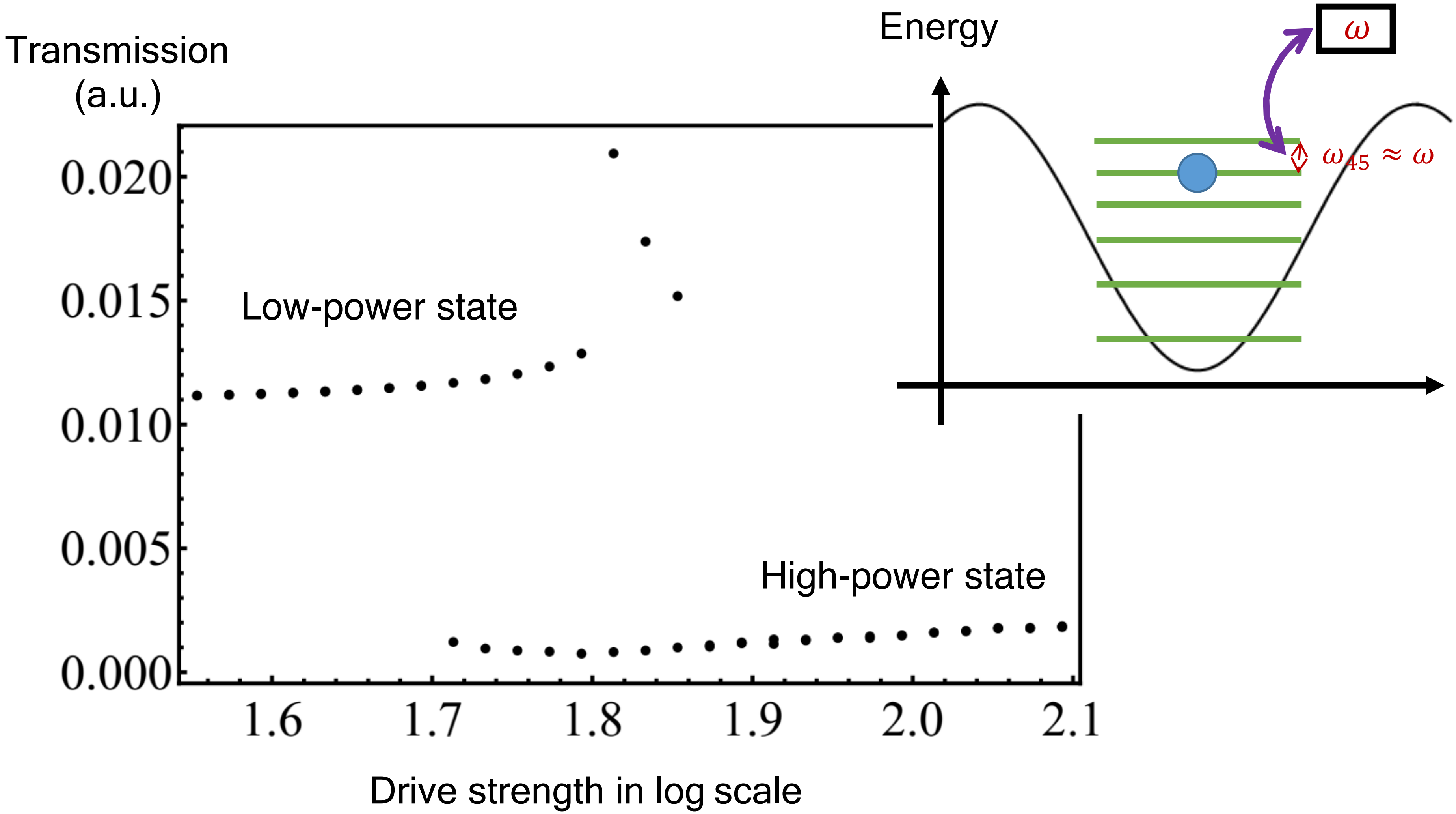


Exp.

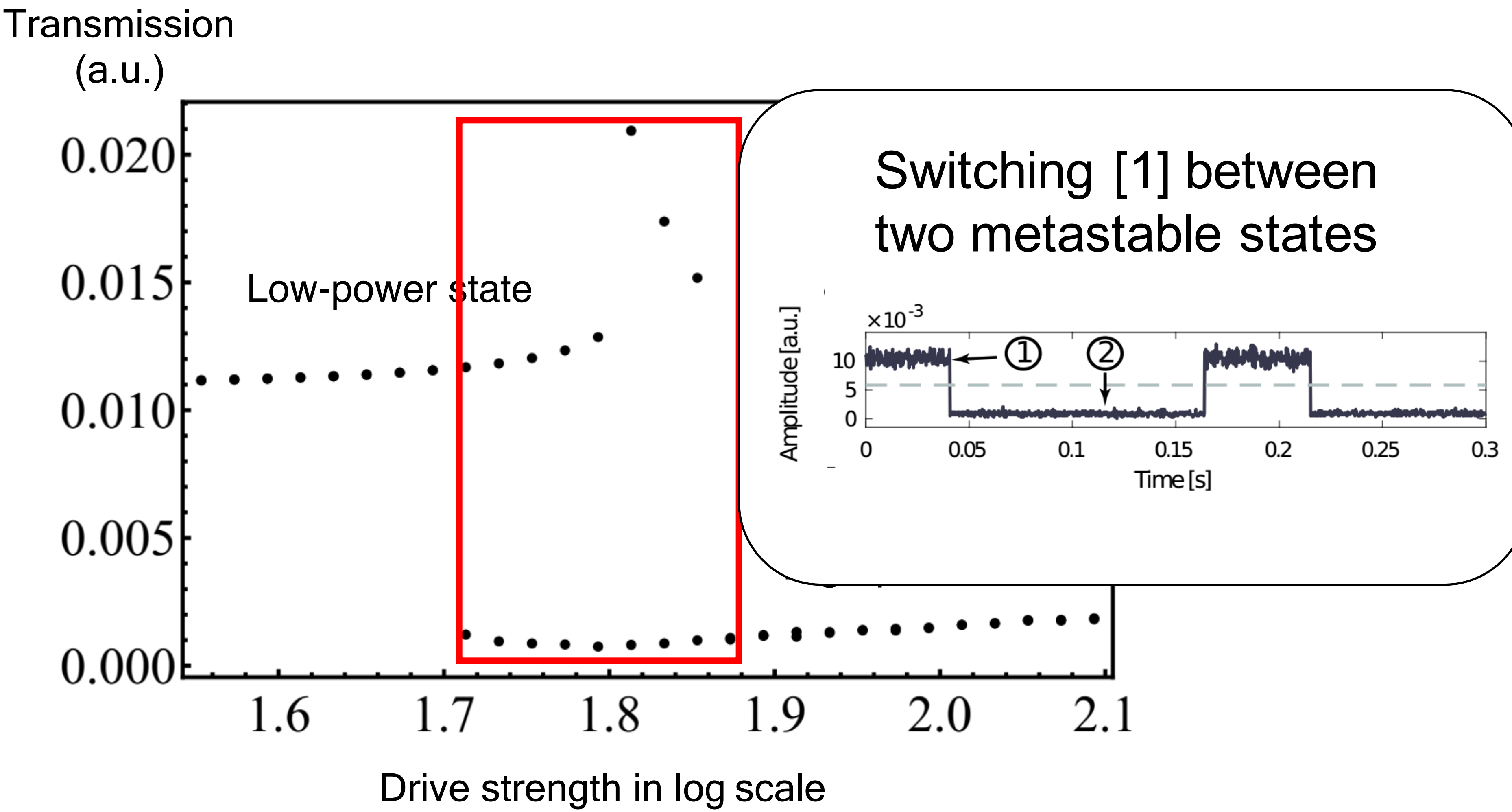


- Low power: resonance peaks ↔ shifted resonator modes
- Increasing power: splitting of low-power resonance peaks
- High power: suppressed transmission
- Sharp crossover: signature of dissipative phase transition

Higher transmon levels



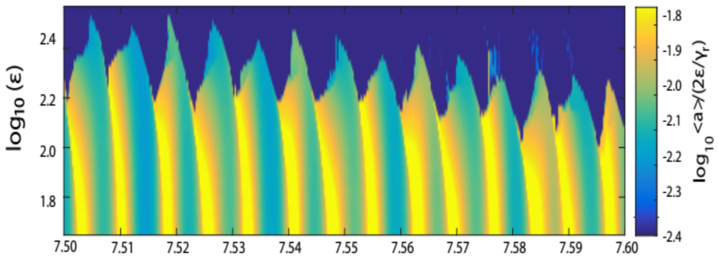
Mean-field bistability



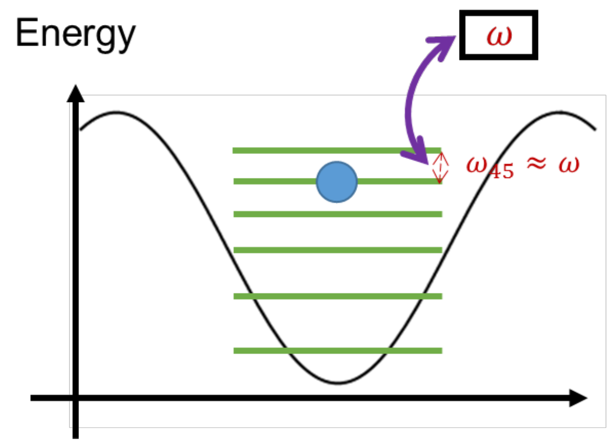
[1] M. Dykman and V. Smelyanskiy, Zh. Eksp. Teor. Fiz 94, 61 (1988)
T. E. Lee, *et al.*, PRL 108, 023602 (2012)

Conclusions

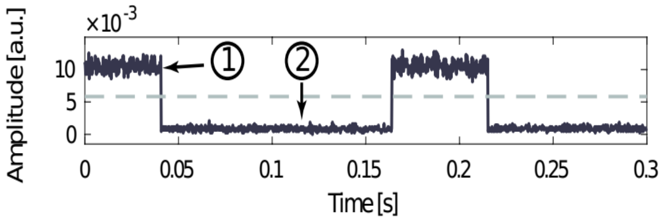
- *First* dissipative phase transition observed in circuit-QED chain



- Rich physics due to *higher transmon levels*



- Future work:
  - High-power state
  - Switching mechanism



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