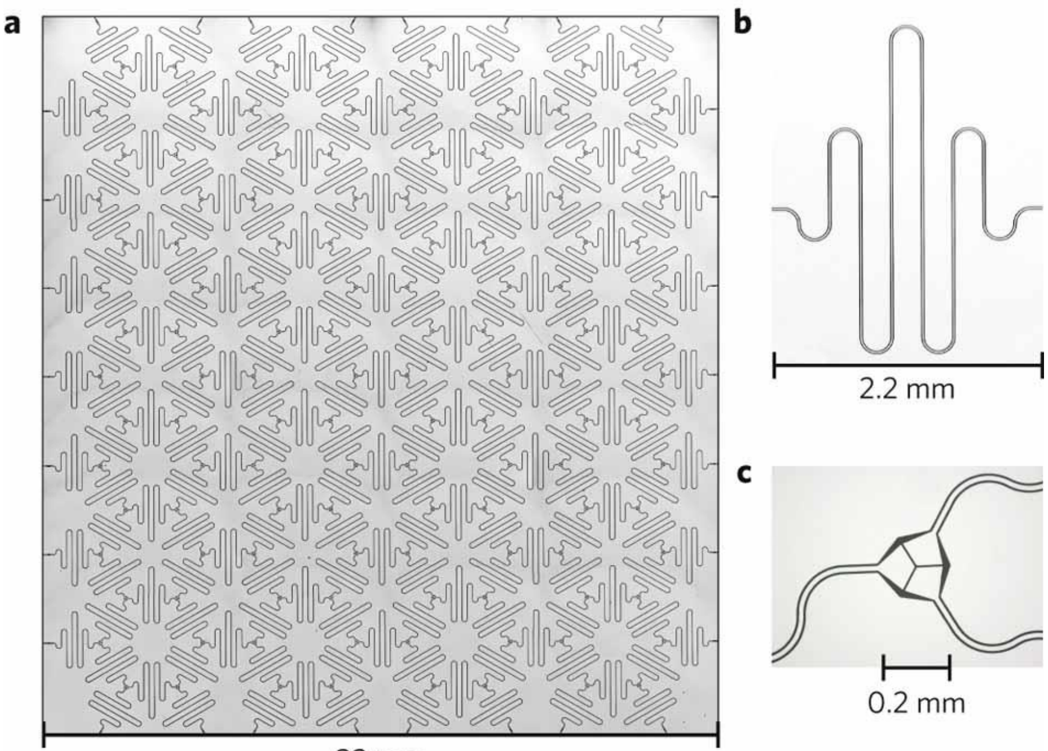
APS March Meeting 2014

Details: arXiv:1311.3227 (2013)

Perturbative approach to open circuit QED systems

Andy C.Y. Li
(Northwestern U)



A. A. Houck *et al*, *Nat Phys* **8,** 292 (2012).

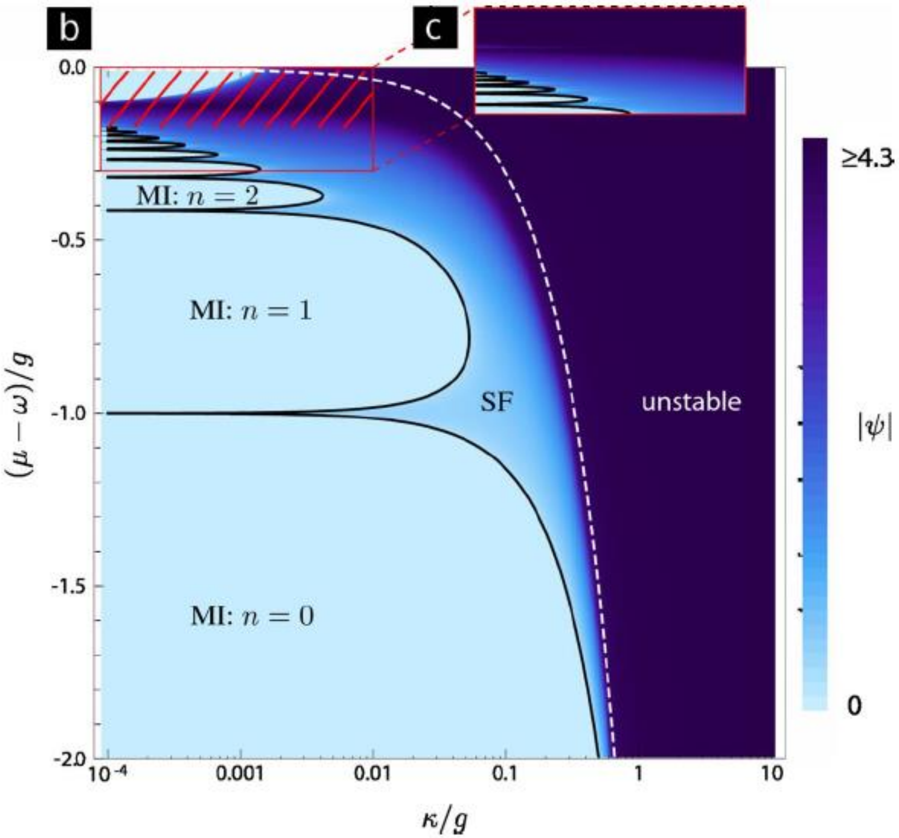
Francesco Petruccione
(U of KwaZulu-Natal)

Jens Koch
(Northwestern U)

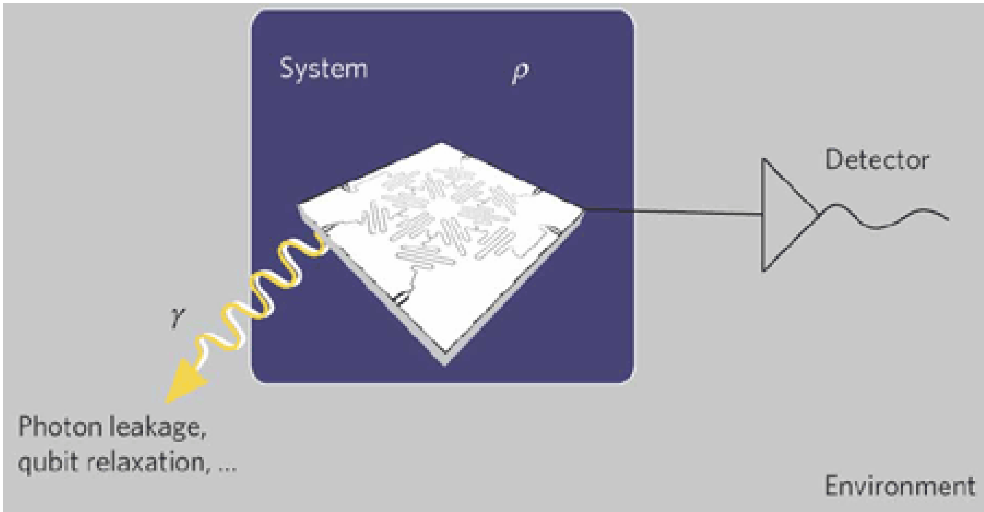
NSF

NORTHWESTERN UNIVERSITY

Motivation – Quantum Simulators



A. A. Houck *et al*, *Nat Phys* **8,** 292 (2012).



<u>Jaynes-Cummings lattice</u>

- Closed systems: Superfluid (SF) to Mott Insulator (MI) phase transition
- Circuit QED systems: zero chemical potential for photon, external drive, photon leakage and qubit relaxation.

=> **Open systems**

Outline

- Perturbation Theory (PT) for Markovian open quantum systems
- PT vs exact result: good agreement

Master equation

- Master equation:

Dissipator

Density matrix

$$\frac{d\rho}{dt} = \mathbb{L}\rho = -i\,[H,\rho] + \mathbb{D}\rho$$

Liouville superoperator (non-Hermitian)

$\because$ $\mathbb{L}$ may not be diagonalizable

- Eigenvalues problem:

$$\mathbb{L}u_\mu = \lambda_\mu u_\mu \qquad\qquad \mathbb{L}^\dagger w_\mu = \lambda_\mu^* w_\mu$$

$\therefore$ u_μ and w_μ may not be complete

- Steady state ρ_{s} : $\dfrac{d\rho_{\mathrm{s}}}{dt} = \mathbb{L}\rho_{\mathrm{s}} = 0$

  -> ρ_{s} : **right eigenstate with eigenvalue zero**

Perturbation Theory (PT)

- Difficult to obtain the eigenvalues and eigenstates

  e.g. 16 spins

  $\mathbb{L}$: 4^{16} X 4^{16} ≈ 4 billion X 4 billion!

- Approximation: PT (analogous to closed-system PT)

  separate the Liouville superoperator

Perturbation

$$\mathbb{L} = \mathbb{L}_0 + \alpha\,\mathbb{L}_1$$

Unperturbed

Small parameter

Previous work on open-system PT

(F Benatti *et al* 2011 *J. Phys. A: Math. Theor.* **44** 155303)

IOP PUBLISHING JOURNAL OF PHYSICS A: MATHEMATICAL AND THEORETICAL

J. Phys. A: Math. Theor. **44** (2011) 155303 (12pp) doi:10.1088/1751-8113/44/15/155303

Asymptotic entanglement and Lindblad dynamics: a perturbative approach

F Benatti[1,2], A Nagy[3] and H Narnhofer[4]

[1] Dipartimento di Fisica Teorica, Università di Trieste, 34014 Trieste, Italy
[2] Istituto Nazionale di Fisica Nucleare, Sezione di Trieste, 34014 Trieste, Italy
[3] Budapest University of Technology and Economics, Muegyetem rkp. 3-9, Hungary
[4] Institut für Theoretische Physik, A-1090 Wien, Austria

E-mail: benatti@ts.infn.it

Received 29 December 2010, in final form 23 February 2011
Published 18 March 2011
Online at stacks.iop.org/JPhysA/44/155303

- Series expansion of the steady state

$$\rho_{\rm s} = \sum_{j\geq 0} \alpha^j \rho_{\rm s}^{(j)}$$

- **Cannot apply to other eigenstates of $\mathbb{L}$**
- **Positivity of $\rho_{\rm s}$ is not mentioned**

Density-matrix PT

- The zeroth-order eigenvalue problem can be solved.

$$\mathbb{L}_0 u_\mu^{(0)} = \lambda_\mu^{(0)} u_\mu^{(0)} \qquad \mathbb{L}_0^\dagger w_\mu^{(0)} = (\lambda_\mu^{(0)})^* w_\mu^{(0)}$$

- Series expansion

$$\boxed{\mathbb{L} = \mathbb{L}_0 + \alpha\,\mathbb{L}_1 \qquad \lambda_\mu = \sum_{j=0}^{\infty} \alpha^j \lambda_\mu^{(j)} \qquad u_\mu = \sum_{j=0}^{\infty} \alpha^j u_\mu^{(j)}}$$

$\Downarrow$ **Substitution**

$$\boxed{\mathbb{L} u_\mu = \lambda_\mu u_\mu}$$

Density-matrix PT

non-invertible and not necessarily diagonalizable

- Group terms of the same order of α

$$\left(\mathbb{L}_0 - \lambda_\mu^{(0)}\right) u_\mu^{(j)} = -\mathbb{L}_1 u_\mu^{(j-1)} + \Delta_\mu^{(j)}$$

inner product

$$\langle x, y \rangle \equiv \mathrm{Tr}\left[x^\dagger y\right]$$

Solve

$$\lambda_\mu^{(j)} = \left\langle w_\mu^{(0)}, \mathbb{L}_1 u_\mu^{(j-1)} \right\rangle - \sum_{k=1}^{j-1} \lambda_\mu^{(k)} \left\langle w_\mu^{(0)}, u_\mu^{(j-k)} \right\rangle$$

$$u_\mu^{(j)} = \left(\mathbb{L}_0 - \lambda_\mu^{(0)}\right)^{-1} \left(-\mathbb{L}_1 u_\mu^{(j-1)} + \Delta_\mu^{(j)}\right)$$

Moore-Penrose pseudoinverse

Steady state: issue of positivity

∵ **probability is non-negative,**
∴ **positive-semidefinite**

- Steady state

$$\rho_{\mathrm{s}} = \sum_{j=0}^{\infty} \alpha^j \rho_{\mathrm{s}}^{(j)}$$

- Truncation to *M*-th order

$$\rho_{\mathrm{s}}^{\mathrm{a};M} = \sum_{j=0}^{M} \alpha^j \rho_{\mathrm{s}}^{(j)}$$

not necessary to be positive

Amplitude-matrix PT

- Since density matrix is positive-semidefinite,

$$\rho = \zeta\zeta^\dagger \longleftarrow \zeta$$ **amplitude matrix**

- If ρ_{s} is positive-definite, ζ_{s} is uniquely defined by Cholesky decomposition and has a series expansion $\zeta_{\mathrm{s}} = \sum_{j=0}^{\infty} \alpha^j \zeta_{\mathrm{s}}^{(j)}$

$$\rho_{\mathrm{s}}^{\mathrm{a};M} = \left(\sum_{j=0}^{M} \alpha^j \zeta_{\mathrm{s}}^{(j)}\right)\left(\sum_{j=0}^{M} \alpha^j \zeta_{\mathrm{s}}^{(j)}\right)^\dagger$$ **is always positive**

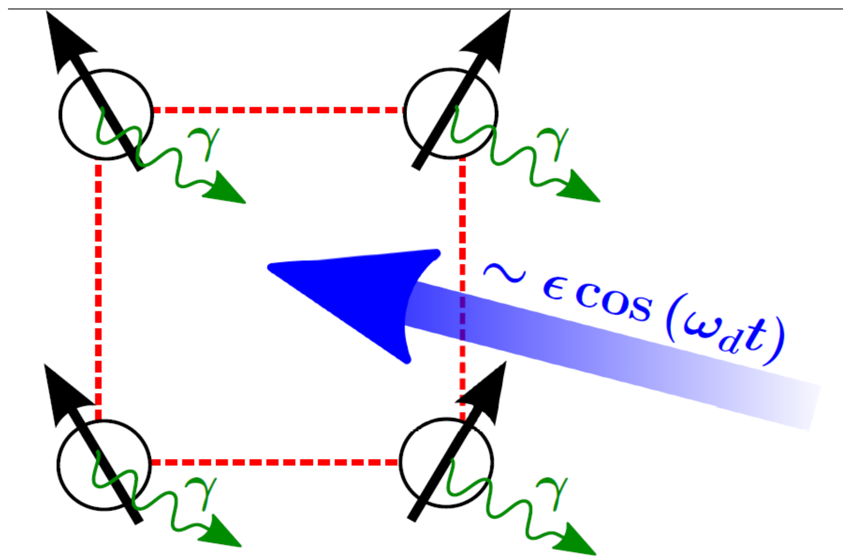
- Determination of each order:

$$\zeta_{\mathrm{s}}^{(0)}\left(\zeta_{\mathrm{s}}^{(0)}\right)^\dagger = \rho_{\mathrm{s}}^{(0)},$$ (Cholesky decomposition)

$$\mathbb{Z}_0\zeta_{\mathrm{s}}^{(j)} = \rho_{\mathrm{s}}^{(j)} - \sum_{k=1}^{j-1} \zeta_{\mathrm{s}}^{(k)}\left(\zeta_{\mathrm{s}}^{(j-k)}\right)^\dagger$$ (Recursive relation)

where $\mathbb{Z}_0 \bullet = \zeta_{\mathrm{s}}^{(0)}(\bullet)^\dagger + \bullet(\zeta_{\mathrm{s}}^{(0)})^\dagger$.

Example



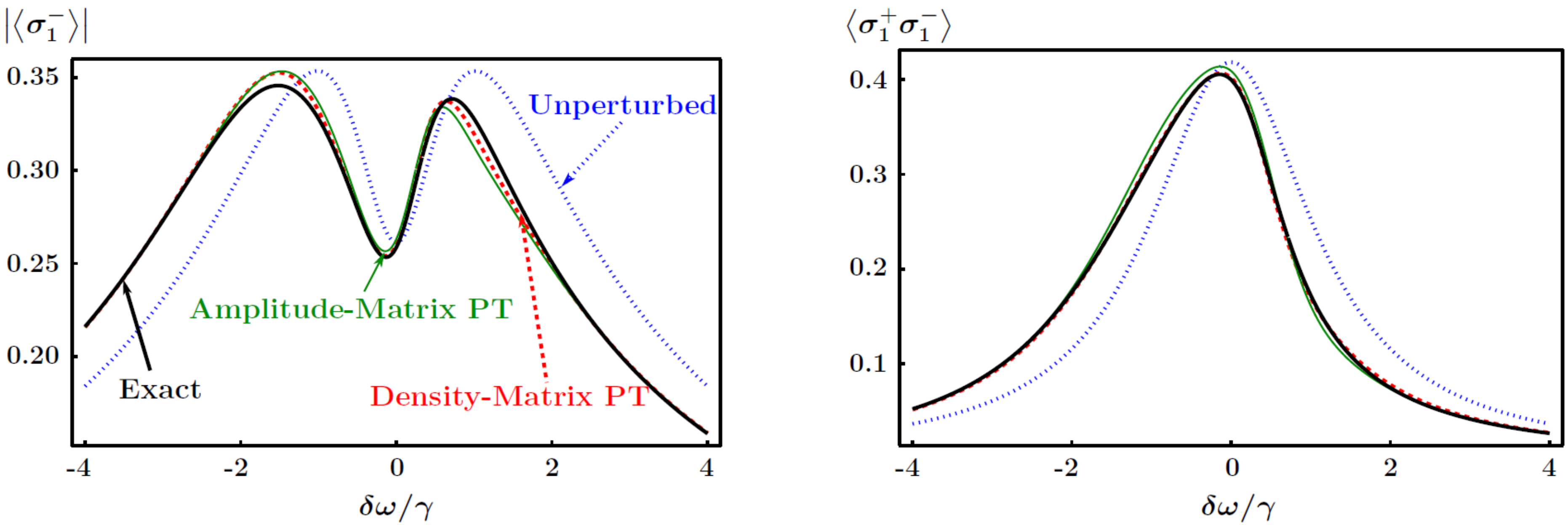
Treated as perturbation

In frame co-rotating with the drive:

$$H = \sum_n \left[\delta\omega\, \sigma_n^+ \sigma_n^- + \epsilon(\sigma_n^+ + \sigma_n^-)\right] + t \sum_n \left(\sigma_n^+ \sigma_{n+1}^- + \text{h.c.}\right)$$

$$\mathbb{L}\rho = -i[H, \rho] + \gamma \sum_n \mathbb{D}\left[\sigma_n^-\right]\rho$$

Second-order PT



Drive strength: $\frac{\epsilon}{\gamma} = 0.8$

Spin-spin coupling strength: $\frac{t}{\gamma} = 0.4$

Summary

- Density matrix PT
  - perturbative correction to eigenvalues and eigenstates of $\mathbb{L}$ (e.g. steady state)
- Amplitude matrix PT
  - tackles the issue of positivity
- Two schemes provide similar accuracy for expectation values
- Outlook: lattice systems
  - systematic hierarchy of coupled clusters of sites