Dissipative phase transition in a one-dimensional circuit QED lattice: Theory

Andy C. Y. Li

Mattias Fitzpatrick
(Princeton U)

Neereja Sundaresan
(Princeton U)

Andrew Houck
(Princeton U)

Jens Koch
(Northwestern U)
1D Circuit-QED chain - model

Resonators coupled to transmons

\[ H_{\text{on-site}} = \sum_{j=1}^{N} \left( \omega a_j^+ a_j + H_{TQ;j} \right) \]

Hopping between resonators

\[ H_{\text{hop}} = \kappa \sum_{j=1}^{N-1} \left( a_j^+ a_{j+1} + a_j a_{j+1}^+ \right) \]

Drive at the input port

\[ H_{\text{drive}} = \varepsilon \left( a_1^+ e^{i\omega_d t} + \text{h.c.} \right) \]

Master equation

\[
\frac{d\rho}{dt} = -i \left[ H_{\text{on-site}} + H_{\text{hop}} + H_{\text{drive}}, \rho \right] + \gamma \sum_{j=1}^{N} \mathbb{D} \left[ a_j \right] \rho + \Gamma \sum_{j=1}^{N} \mathbb{D}_{TQ;j} \rho
\]

Transmon qubit relaxation

photon decay
Transmon $\approx$ Bose-Hubbard

\[
H_{\text{TQ}} = 4E_C \left( \frac{q}{2e} - \frac{q_g}{2e} \right)^2 - E_J \cos \left( \frac{2e}{\hbar} \phi \right)
\]

\[
H_{\text{BH}} = \Omega b^\dagger b + U b^\dagger b^\dagger bb
\]

\[
\omega_{10} = -E_C / 2
\]

\[
\omega_{21} \approx \omega_{10} - E_C
\]

\[
\omega_{10} \approx \sqrt{8E_J E_C}
\]

Jens Koch, et al., PRA 76, 042319 (2007)
Theoretical challenges

• Strong coupling: \( g \approx 500 \text{ MHz} \) (Detuning \( \Delta \approx 2 \text{ GHz} \))

• Drive power over 5 orders of magnitude: dispersive approximation

• Large number of sites: direct numeric

• Critical behavior: perturbative treatment
Mean-field treatment

\[ \frac{d\rho}{dt} = i[H, \rho] + \gamma \sum_{j=1}^{N} \mathcal{D}[a_j] \rho + \Gamma \sum_{j=1}^{N} \mathcal{D}[b_j] \rho \]

\[ H = \sum_{j=1}^{N} \left[ \delta \omega a_j^\dagger a_j + \delta \Omega_j b_j^\dagger b_j + U b_j^\dagger b_j^\dagger b_j b_j + g \left( a_j^\dagger b_j + a_j b_j^\dagger \right) \right] + \sum_{j=1}^{N-1} \kappa \left( a_j^\dagger a_{j+1} + a_j a_{j+1}^\dagger \right) + \epsilon \left( a_1^\dagger + a_1 \right) \]

Mean-field approximation:

\[ a_j = \alpha_j + c_j \quad b_j = \beta_j + d_j \]

Mean-field dynamics equations

\[ i \frac{d\alpha_j}{dt} = \left( \delta \omega - i \frac{\gamma}{2} \right) \alpha_j + g \beta_j + \kappa \left( \alpha_{j-1} + \alpha_{j+1} \right) + \epsilon \delta_{j1} \]

\[ i \frac{d\beta_j}{dt} = \left( \delta \Omega_j - i \frac{\Gamma}{2} \right) \beta_j + g \alpha_j + 2U |\beta_j|^2 \beta_j \]
Transmission: theory vs. exp.

- Low power: resonance peaks ↔ shifted resonator modes
- Increasing power: splitting of low-power resonance peaks
- High power: suppressed transmission
- Sharp crossover: signature of dissipative phase transition
Higher transmon levels

Drive strength in log scale

Transmission (a.u.)

Energy

Low-power state

High-power state

\[ \omega_{45} \approx \omega \]
Mean-field bistability

Transmission (a.u.)

Low-power state

Switching [1] between two metastable states

Conclusions

• *First* dissipative phase transition observed in circuit-QED chain

• Rich physics due to *higher transmon levels*

• Future work:
  • High-power state
  • Switching mechanism